Partitioning of Distributed MIMO Systems based on Overhead Considerations

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***Abstract*—Distributed Multiple Input Multiple Output (MIMO) networks are expected to address the challenges of the enormous traffic demands of future wireless networks. A limiting factor that is directly related with these system performance is the overhead signaling required for distributing data and control information among the network elements. Recently, considering interference alignment-type networks, partitioning has been proposed as an efficient approach for reducing the required overhead, and thus optimizing the network effective sum-rate.**

In this paper, the framework of orthogonal partitioning is extended to distributed MIMO networks employing joint multi-user beamformimg, aiming to maximize the effective sum-rate, either with or without overhead constraints. Moreover, we introduce the formulation of the constrained orthogonal partitioning as an elegant Knapsack optimization problem. In contrast to previous works, a maximum allowed overhead is considered, in order to comply with possible requirements of practical systems, where the overhead subframe size cannot dynamically change. Several numerical results are presented, giving insight into the capabilities of distributed MIMO networks and the actual sum-rate scaling under realistic overhead constraints.

***Index Terms*—Distributed MIMO, knapsack optimization, network partitioning, overhead reduction.**

I. Introduction

Distributed Multiple Input Multiple Output (MIMO) networks (also known as network MIMO) have attracted great research interest for their potential to satisfy the very high data rates demands of future wireless networks. The generic system model comprises a number of distributed access points (AP)s-(transmitters), which communicate with a number of clients (receivers), forming a virtual wireless MIMO array, which mimics a conventional collocated MIMO system. Depending on the ratio between the number of APs and clients as well as the kind of information that is shared among the network elements, several different techniques have been proposed, which aim at interference mitigation or sum-rate scaling. For example, if the network channel matrices are known at the transmitter and receiver sides, interference alignment (IA) may achieve a linear rate scaling with the number of users in a network [1]. Dirty paper coding is another well-known scheme for achieving sum-rate scaling in multi-user networks [2], with, however, implementation difficulties on practical systems. Besides channel state information (CSI), user data can be also shared between the transmitters enabling joint multi-user beamforming, which scales the network rate with the number of transmitting devices [3]. From a theoretical point of view, more than one hop communications, such as hierarchical cooperation schemes, may achieve a linear rate scaling with the number of transmitters [4], also with the cost of user data sharing procedures.

Due to the substantial amount of information that must be shared, among the network components, for performing various operations, including CSI estimation, time and frequency synchronization, data sharing etc, the required overhead is expected to significantly increase with the number of APs and clients [1], [5], [6]. In [6], it was shown that the limiting factor for network MIMO systems is not just the complexity and rate of the backbone wired network, but the intrinsic dimensional limitation of estimating the channels. In this context, the overhead reduction of the distributed MIMO has recently gained increased interest, e.g., [3], [7]–[10]. In [3], based on a new low-overhead technique for synchronizing the phases of the transmitters, a new joint multiuser beamforming (JMB) scheme for independent APs was presented. Considering the downlink of a multicell system with multiantenna base stations and single-antenna users, the trade-off between the benefits of a larger number of cooperating antennas and the cost of estimating higher-dimensional channel vectors has been assessed [7]. In [9] a framework for distributed MIMO ad-hoc networks is proposed aiming to improve link capacity and reduce the interworking with other links. Moreover, in [10], clustering of distributed MIMO networks has been proposed in order to make the processing overhead, such as the computation of precoding matrices, more manageable, which however requires inter-cluster interference mitigation techniques. In a recent work [1], a novel concept was introduced, where a distributed MIMO network employing IA, is partitioned into orthogonal groups (e.g., in a time division multiple access (TDMA) fashion), eliminating in this way any kind of interference in the network. Additionally, the overhead penalty on the sum-rate has been investigated, showing that the effective sum-rate goes to zero as the number of users increases, whilst the orthogonal partitioning is shown to increase the effective sum-rate.

A. Motivation

The partitioning of a distributed MIMO network into orthogonal groups is a very promising concept for maximizing the effective sum-rate. Specifically, orthogonal grouping decreases the MIMO size within each group, and hence the required overhead per partition, but also decreases the spectrum utilization (e.g., due to TDMA), resulting in an interesting trade-off. The partitioning algorithms, presented in [1], target the

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maximization of the effective sum-rate of a network employing IA, assuming that the size of the overhead subframe within the entire frame can dynamically change. Although, this is the optimal strategy in terms of sum-rate maximization, it may not be the case for several practical systems, where only a predetermined portion of the frame is available, e.g., in Long Term Evolution based networks [11]. In this paper, based on the JMB system model [3], we formulate an optimization problem, which selects the optimal partitioning in terms of maximum effective sum-rate, considering two distinct cases, namely unconstrained optimization and constrained optimization, based on the maximum allowed portion of the frame that is available for overhead. Our motivation is to get insight into the maximum achievable effective sum-rate, thus, providing to the network designer the opportunity to compare the optimal (unconstrained) performance with the corresponding one when realistic/practical constraints are imposed by the system frame structure.

B. Contribution

The contribution of this paper is two fold. First, we extend the orthogonal partitioning optimization problem of networks employing IA [1] to the distributed MIMO network, employing the JMB scheme, where the overhead size within the frame is unconstrained. For this case, an exhaustive search approach is employed for the optimal partitioning of the network. Moreover, a new optimization problem is introduced, which takes into account the case where the overhead size within the frame is constrained, in order to comply with possible requirements of practical systems where the overhead size cannot dynamically change [11]. In this context, this partitioning optimization problem is formulated as an elegant Knapsack problem, for the first time, and its exact solution is computed. Several results are presented, giving insight into the capabilities of distributed MIMO networks and the actual sum-rate scaling, when overhead is taken into account.

C. Outline

The remainder of this correspondence is organized as follows: In Section II, the system model is presented together with a brief overview of the zero forcing beamforming (ZFBF) technique. In Section III, both partitioning techniques, namely the optimum (unconstrained) and the suboptimum (with constraints) are presented and their performance is analyzed. In Section IV, numerically evaluated results are presented and discussed, while the concluding remarks can be found in Section V. We note that uppercase boldface letters are used for matrices and lowercase boldface for vectors, while $\mathbf{X}^*$ $(\mathbf{x}^*)$ represents the conjugate transpose of a matrix $\mathbf{X}$ (vector $\mathbf{x}$), and $|A|$ denotes the dimension of A.

II. System Model

The downlink of a distributed MIMO communication scenario is considered, where K distributed transmit antennas (APs) communicate with $M = K$ spatially distributed single-antenna nodes (clients), (see Fig. 1). In this context and towards an interference-free communication scenario, the JMB concept is employed, [3], and the following assumptions are made:

- The APs are interconnected through a high capacity backhaul and have access to the clients data. Investigating the information exchanges via the backhaul is beyond the scope of this work.
- A time-varying fading channel is considered, with the fading amplitudes at each path being Rayleigh distributed, i.e., the probability density function (PDF) is given by
$$f_h(h) = \frac{2h}{\Omega} \exp\left(-\frac{h^2}{\Omega}\right), \ h \leq 0 \tag{1}$$
where $\Omega = \mathbb{E}\langle h^2 \rangle$, with $\mathbb{E}\langle \cdot \rangle$ denoting expectation.
- Transmissions and receptions are synchronous
- Perfect CSI is available at the transmitters
- Error-free, zero delay feedback links from each client to the AP are assumed.
- The APs employ the ZFBF technique to avoid interference among clients.

Let $s_k, \mathbf{h_k}, \mathbf{w_k}, P_k$ denote the data symbol, the channel (row) vector gain, the beamforming column vector and the transmit power allocated to user k. In this context, the beamforming weights $(\mathbf{W} = [\mathbf{w_1}, \mathbf{w_2}, \cdots, \mathbf{w_K}])$ are appropriately selected in order to satisfy the zero-interference condition, that

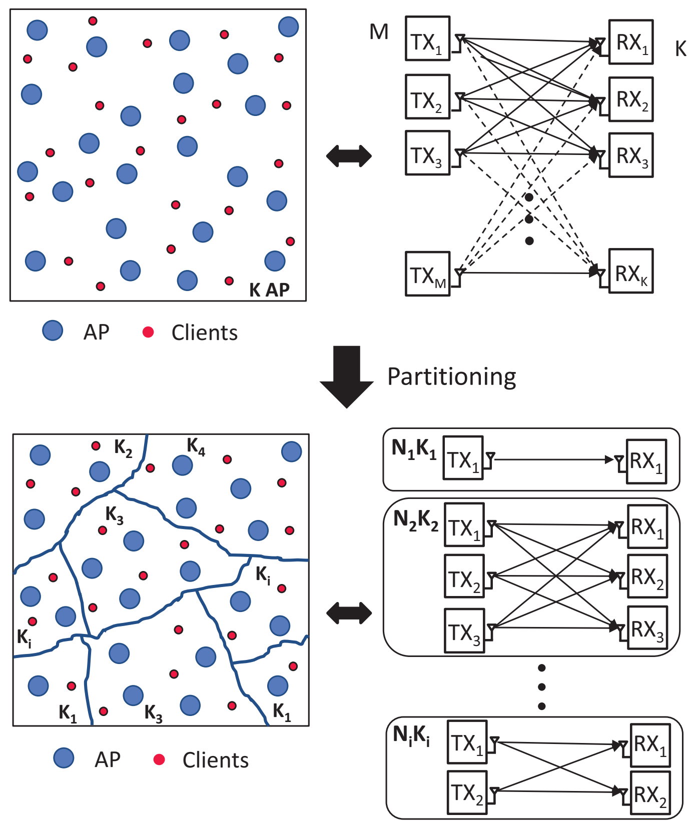

Fig. 1. A distributed MIMO network, with JMB for interference elimination. The required overhead scales with the size of MIMO, employing the JMB. The entire MIMO network is partitioned into orthogonal groups (e.g., TDMA partitioning), each with a reduced MIMO size and hence reduced overhead. In all partitions, the number of APs is the equal to the number of clients and JMB is employed.

is $\mathbf{h_k w_j} = 0$ for $k \neq j$. The zero-interference can be obtained using the pseudoinverse of the channel gain matrix as weights, so that the received signal can be written as

$$y_k = \left(\sqrt{P_k}\mathbf{w_k h_k}\right) s_k + \sum_{j\neq k}\left(\sqrt{P_j}\mathbf{w_j h_j}\right) s_j + z_k,\ k = 1,\ldots,K \tag{2}$$

where z_k is the additive white Gaussian noise (AWGN) at the kth user, while y_k is the received signal by the kth user. By adopting ZFBF, the sum term in (2) is considered to be zero. Furthermore, the sum-rate of the ZFBF can be expressed as

$$R_{ZFBF} = \sum_{i=1}^{K} \log_2\left(1 + P_i\right). \tag{3}$$

The optimal P_i can be obtained as $P_i = (\mu\gamma_i - 1)^+$, with x^+ denoting $\max(x, 0)$, and μ satisfying

$$\sum_{i=1}^{K}\left(\mu - \frac{1}{\gamma_i}\right) = P. \tag{4}$$

In (4), γ_i is the effective channel gain to the ith user [12], defined as

$$\gamma_i = \frac{1}{||\mathbf{w_i}||^2} \tag{5}$$

and P represents the power constraint for the distributed MIMO scheme considered.

III. Partitioning Optimization to Reduce the Overhead

The distributed MIMO schemes, such as the JMB one, can in principle increase the sum-rate of a network. Practically though, due to the required overhead, the effective sum-rate (i.e., the actual transmitted data) may significantly decrease as the MIMO network size increases. Towards maximizing the effective sum-rate, partitioning the network into orthogonal JMB groups (e.g., in terms of TDMA), each with a reduced MIMO size, has been proved a promising approach (see Fig. 1). Using orthogonal groups decreases the MIMO size, and hence the required overhead per partition, but also decreases the spectrum utilization, resulting in an interesting trade-off. In this section, we present the optimal partitioning of the JMB scheme based on an exhaustive search on all possible partitioning combinations and a Knapsack-based partitioning, where the maximum overhead portion within a frame is constrained.

Similar to [1], we assume that the overhead includes symbols required for training, feedback, synchronization, or any other spectrum utilization not used for communication of data. It is thus a function of the number of clients in the channel and requires a portion of the frame equal to $\alpha = \min\left[\mathcal{L}(K)/T, 1\right]$, where $\mathcal{L}(K)$ is the overhead scaling function with respect to the MIMO partitioning size and T is the frame duration, which is usually less than or equal to the channel coherence time (CCT). The overhead scaling of a D partition network, with $\mathcal{K}_d$ APs and clients, is equal to $\mathcal{L}(\mathcal{K}_d) = \mathcal{K}_d^{\tau}$. More specifically, following the approach proposed in [1], [13], the clients send F_d bits to all the APs, in order to construct the corresponding $\mathbf{w_i}$. Hence, a total of KF_d is broadcasted by each client and thus each AP receives a total of K^2F_d bits of feedback from all the clients.

A. Optimal Partitioning

Considering clients that are partitioned in D index sets ($\mathcal{K}_d$), with $|\mathcal{K}_d|$ clients in the dth group, and the overhead model in [1], the optimal solution aims to find the partitioning combination that maximizes the total sum-rate, i.e.,

$$\begin{aligned} &\textit{maximize} \sum_{d=1}^{K} N_d\,\overline{\alpha}_d R_{ZFBF,d} \\ &\textit{subject to} \sum_{d=1}^{K} N_d|\mathcal{K}_d| = K \end{aligned} \tag{6}$$

where

$$\overline{\alpha}_d = 1 - \alpha_d = \frac{1}{D} - \frac{\mathcal{K}_d}{T}.$$

In (6), $\overline{\alpha}_d$ and α_d represent the portion of the frame of the dth type partition used only for data and overhead, respectively, and N_d represents the total number of dth type partitions that have been generated. For example, when $K = 4$, the optimization in (6) will aim to find the maximum sum-rate among the partitioning combinations $[(4), (3, 1), (2, 2), (2, 1, 1), (1, 1, 1, 1)]$. We solve this problem by exhaustive search among these combinations. The results obtained serve as a benchmark for the suboptimal overhead constrained partitioning, which will be presented in the next subsection. We note that for identically distributed channels among the APs and the clients, it does not matter which particular APs and clients are involved in the partitions. On the contrary, if the channels are not identically distributed, the outcome depends on which APs are included, which also results in a significant increase of the different combinations and hence the complexity of the exhaustive search [1].

B. Overhead-Constrained Partitioning and the Knapsack Algorithm

In this section, we consider the more realistic case, where a maximum allowed overhead size is taken into consideration. Thus, the overhead portion of the frame, is less or equal to a predefined threshold $\alpha_{\rm th}$, i.e., $\sum_{d=1}^{K}\alpha_d < \alpha_{\rm th}$, yielding the following optimization problem

$$\begin{aligned} &\textit{maximize} \sum_{d=1}^{K} N_d\,\overline{\alpha}_d R_{ZFBF,d}(S) \\ &\textit{subject to} \sum_{d=1}^{K} N_d|\mathcal{K}_d| = K \textit{ and } \sum_{d=1}^{K}\alpha_d < \alpha_{\rm th}. \end{aligned} \tag{7}$$

The optimization problem appearing in (7) is known as the bounded Knapsack problem (BKP) [14]. More specifically, given n item types and a knapsack with p_j, w_j and b_j being the profit, the weight and the upper bound on the availability of an item j, respectively, and c the capacity of the knapsack, select a number $x_j (j = 1, \ldots, n)$ of items of each type so as

TABLE I
KNAPSACK TRANSFORMATION ALGORITHM

BKP to zero-one transformation	Sum Rates and Overhead of Basic MIMO Elements
input: $n, p_j, b_j \left(K, R_{ZFBF,i}\right.$[1]$\left., \lfloor\frac{K}{i}\rfloor\right)$	$\overline{p}_1 := \hat{p}_{\hat{n}}\hat{w}_{\hat{q}_{\hat{n}},\hat{r}_{\hat{n}}}$[2]
output: $\overline{v}, \overline{p}_j, \overline{w}_j$	$\overline{w}_1 := \hat{w}_{\hat{q}_{\hat{n}},\hat{r}_{\hat{n}}}$
$\hat{n} := 0$; $\overline{v} := 1$	$\overline{v} := \overline{v} + 1$
begin	**repeat**
for $j := 1$ to n **do**	$\hat{n} := \hat{n} - 1$
begin	**for** $j := 1$ to $\hat{n} - 1$ **do**
$k := 0$	**if** $\hat{m}_{\hat{n}} + \hat{m}_{\hat{n}-j} = n$
repeat	$\overline{p}_{\overline{v}} := \hat{p}_{\hat{n}}\hat{w}_{f(q_1)^3,\hat{r}_{\hat{n}}} + \hat{p}_{\hat{n}-j}\hat{w}_{f(q_1),\hat{r}_{\hat{n}-j}}$
$\hat{n} := \hat{n} + 1$	$\overline{w}_{\overline{v}} := \hat{w}_{f(q_1),\hat{r}_{\hat{n}}} + \hat{q}_{\hat{n}-j}\hat{w}_{f(q_1),\hat{r}_{\hat{n}-j}}$
$\hat{p}_{\hat{n}} := (k+1)p_j$	$\overline{v} := \overline{v} + 1$
$k := k + 1$	**else if** $\hat{m}_{\hat{n}} + \hat{m}_{\hat{n}-j} < n$
$\hat{m}_{\hat{n}} := kj$	**for** $\ell := 1$ to $\hat{n} - 1$ **do**
$\hat{q}_{\hat{n}} := k$	**if** $\ell \neq \hat{n} - j$ & $\hat{m}_{\hat{n}} + \hat{m}_{\hat{n}-j} + \hat{m}_{\ell} = n$
$\hat{r}_{\hat{n}} := j$	$\overline{p}_{\overline{v}} := \hat{p}_{\hat{n}}\hat{w}_{f(q_2)^4,\hat{r}_{\hat{n}}} + \hat{p}_{\hat{n}-j}\hat{w}_{f(q_2),\hat{r}_{\hat{n}-j}} + \hat{p}_{\ell}\hat{w}_{f(q_2),\hat{r}_{\ell}}$
until $k = b_j$	$\overline{w}_{\overline{v}} := \hat{w}_{f(q_2),\hat{r}_{\hat{n}}} + \hat{q}_{\hat{n}-j}\hat{w}_{f(q_2),\hat{r}_{\hat{n}-j}} + \hat{q}_{\ell}\hat{w}_{f(q_2),\hat{r}_{\ell}}$
end	$\overline{v} := \overline{v} + 1$
end	**end**
	end
	end
	end
	until $\hat{n} = 2$

[1] $R_{ZFBF,i}$ represents the sum-rate of the ith order MIMO

[2] $\hat{w}_{x,y}$ is the overhead factor of the yth MIMO order in a x partition

[3] $f(q_1) = \hat{q}_k + \hat{q}_{k-j}$.

[4] $f(q_2) = \hat{q}_k + \hat{q}_{k-j} + \hat{q}_{\ell}$.

to maximize $z = \sum_{j=1}^{n} p_j x_j$, subject to $\sum_{j=1}^{n} w_j x_j \leq c$. It should be noted that the BKP is a generalized formulation of the zero-one knapsack problem and can be simplified to the latter using the algorithm presented in the first column of Table. I. In our case n is equal to K and p_j is equal to $R_{ZFBF,j}$. Following this algorithm our BKP is transformed to

$$\begin{aligned} &\textit{maximize} \ \sum_{i=1}^{\hat{n}} \hat{p}_i x_i \\ &\textit{subject to} \ \sum_{i=1}^{\hat{n}} \hat{w}_i x_i < \alpha_{\text{th}} \ \text{and} \ \sum_{i=1}^{\hat{n}} \hat{m}_i = K \end{aligned} \tag{8}$$

where $\hat{p}_i$ and $\hat{w}_i$ represent the sum-rate (or the overhead) of the ith MIMO basic element. For example for $K = 4$ we have 8 different basic elements, namely $1\text{x}1, 2*(1\text{x}1), 3*(1\text{x}1), 4*(1\text{x}1), 2\text{x}2, 2*(2\text{x}2), 3\text{x}3, 4\text{x}4$. In our case, by using different combinations of these basic MIMO elements our objective is to obtain the optimal network partitioning that maximizes the effective sum-rate, subject to a predefined threshold for the maximum allowed overhead. Hence, firstly, all the different combinations that can be supported by the MIMO scheme under investigation should be exploited. A general solution for this problem represents a cumbersome task. However, we have managed to obtain a representative algorithm that computes the effective sum-rate and the corresponding overhead of all MIMO partitions that a MIMO of maximum 9th order can be divided to (see the second column of Table I). Therefore, following these two algorithms, the initial BKP has been transformed to the following simplified zero-one Knapsack

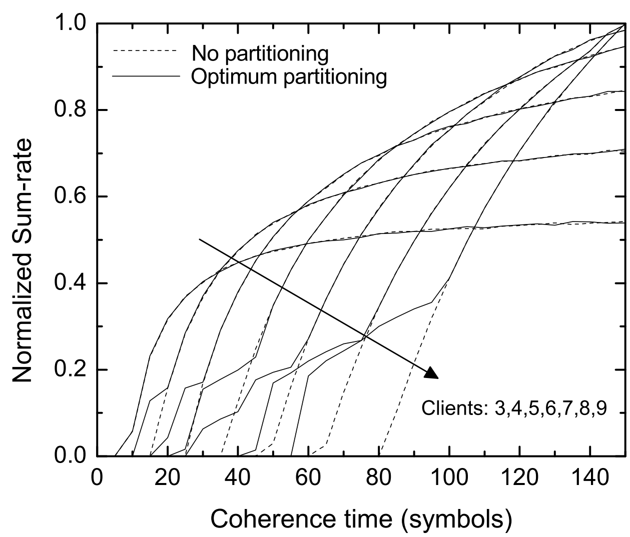

Fig. 2. The NSR with and without network partitioning. The benefits of orthogonal partitioning are visible for higher CCT and larger MIMO dimensions. The sum-rate is normalized by the corresponding performance of the optimal partitioning of a 9x9 MIMO network with SNR=25dB.

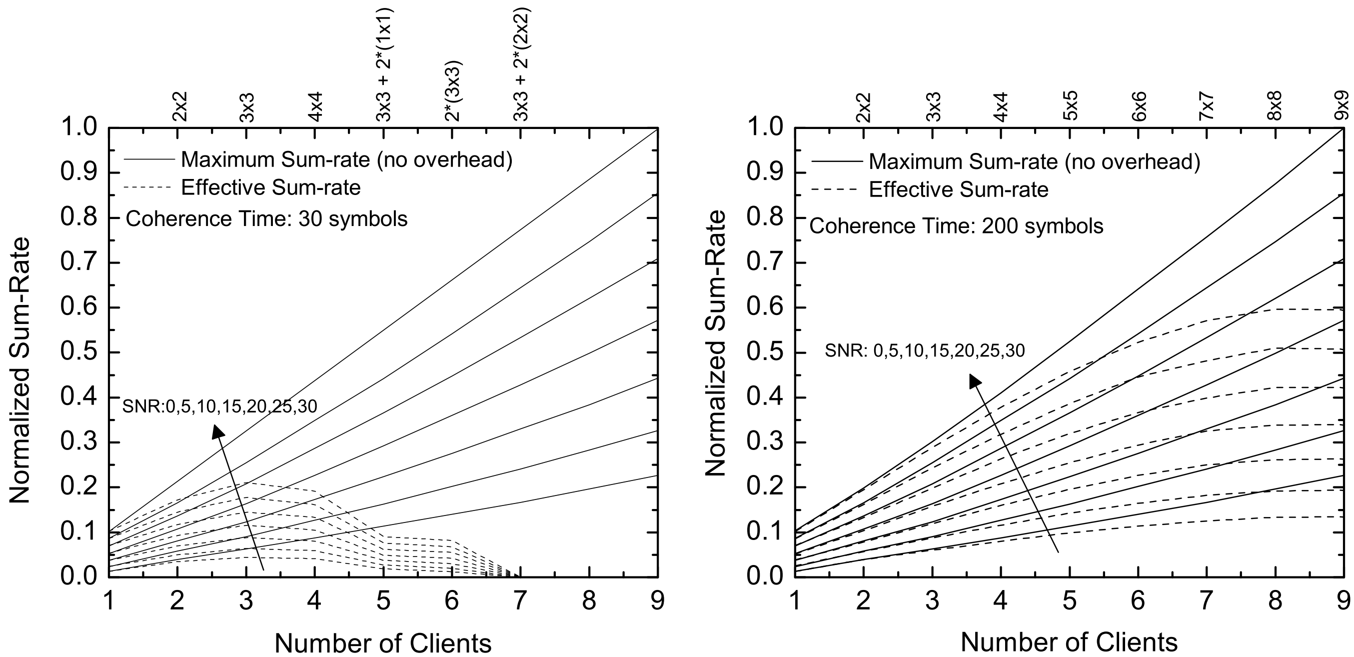

Fig. 3. The maximum achievable sum-rate (ideal case with no overhead) and the effective sum-rate (actual data transmissions) with optimal partitioning vs the number of APs. The sum-rates are normalized by that achieved by the $9\text{x}9$ scheme for SNR = 25dB and zero overhead.

problem

$$\begin{aligned} &\textit{maximize} \ \sum_{i=1}^{\overline{v}} \overline{p}_i x_i \\ &\textit{subject to} \ \sum_{i=1}^{\overline{v}} \overline{w}_i x_i < \alpha_{\text{th}}. \end{aligned} \tag{9}$$

The solution to this zero-one Knapsack problem is a straightforward procedure and the exact optimal solution coincides with the one provided via the Greedy-Split algorithm [15]. Specifically, the following steps are followed

- $\overline{p}_i$ $(i \in \overline{v})$ are sorted on descending order, i.e., $\overline{p}_1 = \max(\overline{p}_i)$
- Examining $\overline{p}_i, \overline{w}_i$ in ascending order
- if $\overline{w}_i < \alpha_{\text{th}}$
- solution$\rightarrow \overline{p}_i, \overline{w}_i$.

Here it is important to note that maximum number of searches is for a $9\text{x}9$ MIMO system is less than 60, hence the complexity of the proposed approach is relatively low.

IV. Performance Results and Discussion

In this section, selected numerical performance evaluation results are presented and discussed. These results include performance comparisons, in terms of the effective sum-rate, for various communication scenarios. The parameters considered in all cases are: randomly deployed single-antenna APs as well as clients, and independent and identically distributed Rayleigh fading channels. It should be noted that for TDMA, overhead is assumed to scale linearly with the number of users.

The benefits of partitioning a distributed MIMO network into orthogonal groups are illustrated in Fig. 2. In this figure, it can be easily observed that partitioning improves the effective sum-rate of the network as the coherence time decreases, while the sum-rate gains become more obvious for larger MIMO networks. As the coherence time increases employing JMB for the whole MIMO network becomes more practical.

In Fig. 3, the normalized sum-rate (NSR) is plotted as a function of the number of the APs for various values of SNR and CCT. The NSR has been evaluated for both cases of maximum achievable sum-rate (ideal case without considering the overhead) and effective sum-rate (with optimal partitioning). In all cases in this figure, the performance improves as the SNR or the number of APs increases. Considering the effective NSR it is important to note that the linear scaling of the sum-rate is not maintained as the number of APs increases, due to the overhead, whilst the performance considerably improves as CCT increases. It should be also mentioned that the partition types change as the CCT increases, as it is depicted at the top axis of all figures. For example, the notation $3\text{x}3 + 2 * (1\text{x}1)$ means that the $5\text{x}5$ MIMO network is partitioned into one partition employing a $3\text{x}3$ MIMO and 2 partitions with a $1\text{x}1$ Single Input Single Output (SISO) scheme.

Considering the constrained algorithm presented in Section III-B, Fig. 4, illustrates the network partitioning as a function of the maximum allowed overhead (as a percentage of the frame duration) and the CCT. In this figure, it is clearly observed that as CCT increases, the size of the MIMO modes, inside the optimal partitions, increases. Hence, for low values of the maximum allowed overhead (MAO) percentage and/or CCT, the optimal solution tends to the multiple SISO partitioning, while as CCT and/or MAO increase, the optimal solution approaches that of a unpartitioned MIMO network.

Finally, Fig. 5 depicts the sum-rate of the constrained sub-optimal partitioning, as a percentage of the sum-rate achieved with optimal partitioning, for various values of the MAO.

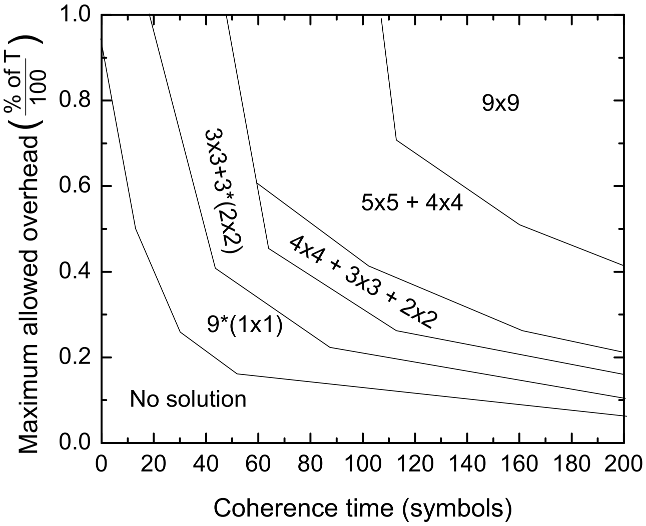

Fig. 4. The constrained partitioning of the network as a function of the MAO and the CCT, for $K = 9$. The optimal solution ranges between the multiple SISO partitioning and the unpartitioned MIMO network.

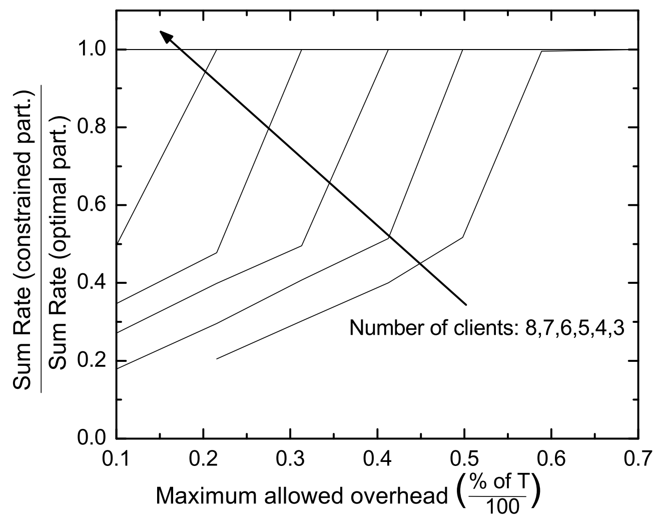

Fig. 5. The suboptimal partitioning sum-rate, as a percentage of the sum-rate achieved with the optimal partitioning vs MAO.

This figure compares the optimal and suboptimal partitioning schemes proposed in Section III, in order to highlight the performance degradation due to the constrained overhead length within the frame. In this figure, it can be verified that for low values of the MAO, the overhead-constrained sum-rate is relatively small compared to the unconstrained one. As MAO increases, the relative performance between these two metrics also increases. Finally, it is noted that for small networks dimensions, the performance of the constrained sum-rate is approaching that of the optimal partitioning.

V. Conclusions

In this correspondence, new partitioning approaches for maximizing the effective sum-rate of distributed MIMO networks are proposed. Firstly, considering unconstrained overhead size, the optimal effective sum-rate of distributed MIMO is presented, while for the second case a maximum allowed overhead size is taken into account, in order to comply with possible requirements of practical systems. For the first approach exhaustive search is employed, while for the second one, the partitioning problem is formulated as a elegant Knapsack optimization problem. Selected evaluated results show that linear scaling of the effective sum-rate is not always maintained as the number of APs increases, due to the overhead required. Relaxing the constraint for fixed overhead size within the frame (i.e., allowing the overhead length to change dynamically) results in performance improvements. Finally, relaxing the assumptions for the orthogonality among the partitions originates an interesting investigation field, related also to more practical considerations, that is going to be included in our future research activities.

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